LOGARITHMIC DENSITY DEPENDENCE OF VISCOSITY AND THERMAL CONDUCTIVITY OF A DENSE GAS

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Polynomial and logarithmic dependences on density of the viscosity and thermal conductivity of a dense gas are compared with experimental data.

In a number of investigations on the kinetic theory of a dense gas ([1-3] and others), it is shown that the coefficients of viscosity and thermal conductivity depend both on the density of the gas and its logarithm:

$$\eta = \eta_0 + \eta_1\rho + \eta_2'\rho^2 \ln\rho + \eta_2''\rho^2 + \ldots, \tag{1}$$

$$\lambda = \lambda_0 + \lambda_1\rho + \lambda_2'\rho^2 \ln\rho + \lambda_2''\rho^2 + \ldots. \tag{2}$$

The form of subsequent terms in Eqs. (1) and (2) has not been established, although it has been suggested [4] that they contain increasing powers of ρ and $\ln \rho$.

The feasibility of approximating experimental data by means of Eqs. (1) and (2) has been investigated by a number of authors. Hanley et al. [5] established from thermal conductivity data for neon that the coefficient λ_1 in Eq. (2) agrees within the limits of its determination with the corresponding coefficient of the linear equation constructed from data for small densities. This fact, in the opinion of these authors, testifies in favor of relationship (2). Le Neindre [6] and Tufeu [7] showed, on the basis of their own experimental data on the thermal conductivity of the inert gases, that the virial coefficients of the thermal conductivity plotted as functions of density are more stable for a logarithmic dependence than for a polynomial.

On the other hand, Gracki and co-workers [8] consider that polynomials are more suitable for the description of their experimental data on the viscosity of helium, argon, hydrogen, and nitrogen. Kestin et al. [9] measured the viscosity of helium, argon, and nitrogen with high accuracy (±0.2%) and investigated its dependence on density. They concluded that the insignificant contribution of the term $\eta_2'\rho^2 \ln \rho$ could be neglected. It can be seen from the tables cited in [9] that the coefficient η_1 changes substantially on going from a linear to a logarithmic dependence.

We have carried out a least-squares analysis of data on the viscosity of neon on the isotherms 25, 50, and 75°C [10] and data on the thermal conductivity of argon at temperatures 0, 25, 50, and 75°C for both polynomial and logarithmic dependences on density. The experimental points were weighted with the values of $1/\eta^2$ and $1/\lambda^2$, which is equivalent to supposing the same relative error of the data on an isothermal. The range of densities was gradually extended in the calculations by including additional points, and when necessary the number of terms of the approximating relationship was increased. This number was determined using the Fisher criterion [12] at a significance level of 0.05. The dimensionalities of thermal conductivity and viscosity used in the description of the experimental data were: λ [10^{-6} kW/(m•deg)], and η [10^{-6} p•sec]; the critical densities used in going over to the reduced density $\omega = \rho/\rho_{cr}$ were: 536.79 Amagat for neon, and 300.38 Amagat for argon.

Figure 1 shows the values of η_0, η_1, η_2, and η_2' for neon on the 75°C isotherm, and Fig. 2 shows λ_0, λ_1, λ_2, and λ_2' for argon on the 0° isotherm for different numbers of terms of the polynomial or logarithmic dependence and different amounts of used data. The magnitudes

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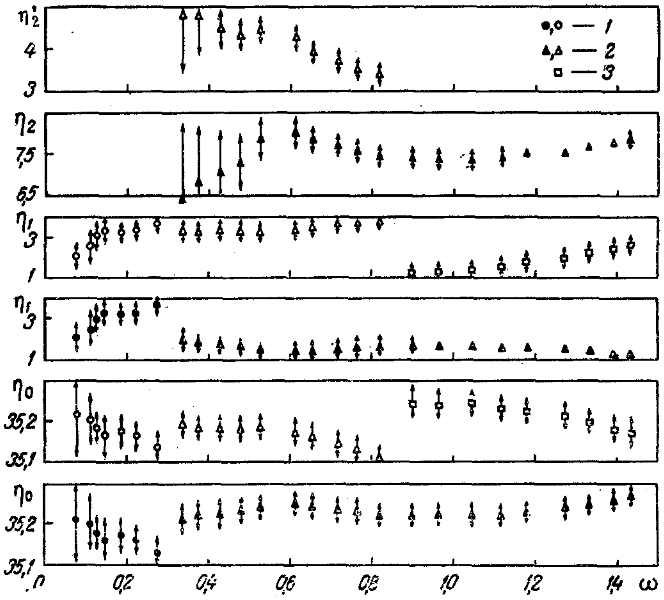

Fig. 1. Dependence of coefficients η_0, η_1, η_2, and η_2' for neon on reduced density on 75°C isotherm for various numbers of terms in the equation. Points 1 correspond to a linear equation; points 2 correspond to a second-degree polynomial and Eq. (1) with three terms; points 3 correspond to Eq. (1) with four terms. The solid points represent polynomial coefficients; the open points represent the coefficients of Eq. (1).

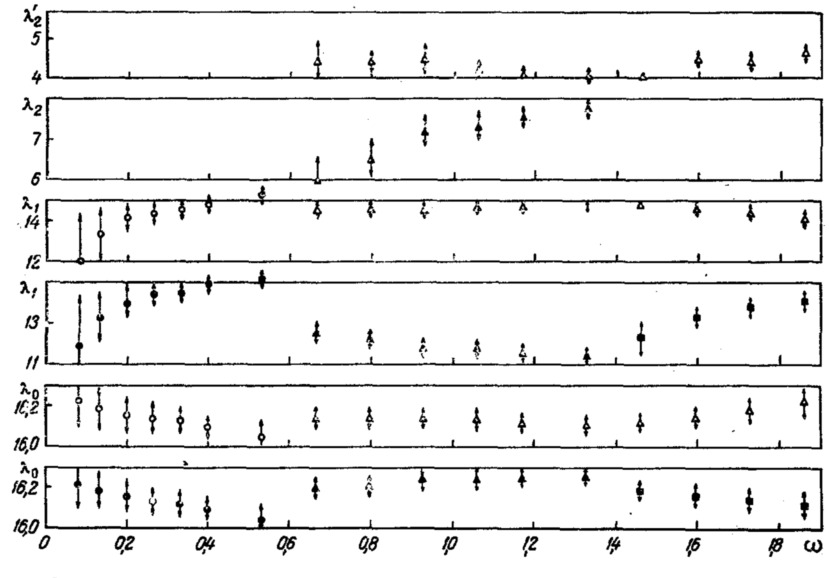

Fig. 2. Dependence of coefficients λ_0, λ_1, λ_2, and λ_2' for argon on reduced density on 0°C isotherm for various numbers of terms in the equation. The notation is as in Fig. 1.

of the errors in the coefficients are denoted by the arrows; the limits within which equations with different numbers of terms operate can be seen due to the different representations of the points. In order to describe the viscosity data on the 75°C isothermal over the whole range of densities (ω = 0-1.44) and on the 25 and 50°C isotherms up to a density ω = 1.3, it is sufficient to utilize a second-degree polynomial. Equation (1) with this same number of terms is valid in the interval ω = 0-0.82 for all temperatures and represents the data with a mean-square error $\delta\eta_{mn}$ = 0.14%, while for a polynomial in this density interval $\delta\eta_{mn}$ = 0.1%. The values of η_2' for ω > 0.82 are determined with a large error and are not shown in Fig. 1.

The thermal-conductivity data on all isotherms can be depicted sufficiently well in the interval ω = 0-1.6 (on the 0°C isotherm up to ω = 1.86) by means of Eq. (2) with three terms. A second-degree polynomial works at all temperatures and in the interval ω = 0-1.3, and describes the experimental data with a mean-square error $\delta\lambda_{mn}$ = 0.6% [the error with Eq. (2) is $\delta\lambda_{mn}$ = 0.5%]. The values of λ_2 outside this interval have a greater error and are not shown in Fig. 2. It can be seen from the figure that the coefficient λ_1 of the logarithmic dependence is more stable than that of the polynomial dependence, and that the values of λ_1 of dependence (2) and of the linear equation are close (allowing for their errors). The picture is not quite so clear for the viscosity: the coefficient η_1 of the polynomials is quite stable over a wide range of densities, but the values of η_1 for the logarithmic dependence differ less from the values of this coefficient for the linear equation.

It can be concluded from an analysis of the results that for the viscosity an ordinary polynomial gives a more accurate representation of the experimental data and works over a wider range of densities than Eq. (1) with the same number of terms. For the thermal conductivity a logarithmic dependence is to be preferred, both from the point of view of the accuracy of approximation of the data and the range in which the equation works, and also from the point of view of the reliability of determination of the lowest virial coefficients on the basis of known criteria [5].

NOTATION

η, λ, viscosity and thermal conductivity of dense gas; η_0, η_1, ..., λ_0, λ_1, ..., virial coefficients of viscosity and thermal conductivity; ρ, density; ρ_{cr}, critical density; $\omega = \rho/\rho_{cr}$, reduced density; $\delta\eta_{mn}$, $\delta\lambda_{mn}$, mean-square deviations of calculated values of η and λ from experimental values.

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